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90639



For Supervisor's use only

Level 3 Calculus, 2007

90639 Sketch graphs of conic sections and write equations related to conic sections

Credits: Three 2.00 pm Thursday 22 November 2007

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables Booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–14 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only Achievement Criteria					
Achievement	Achievement with Merit	Achievement with Excellence			
Sketch graphs of conic sections and write equations related to conic sections.	Solve problems involving conic sections.	Solve more complex conic section problems.			
Overall Level of Performance					

You are advised to spend 30 minutes answering the questions in this booklet.

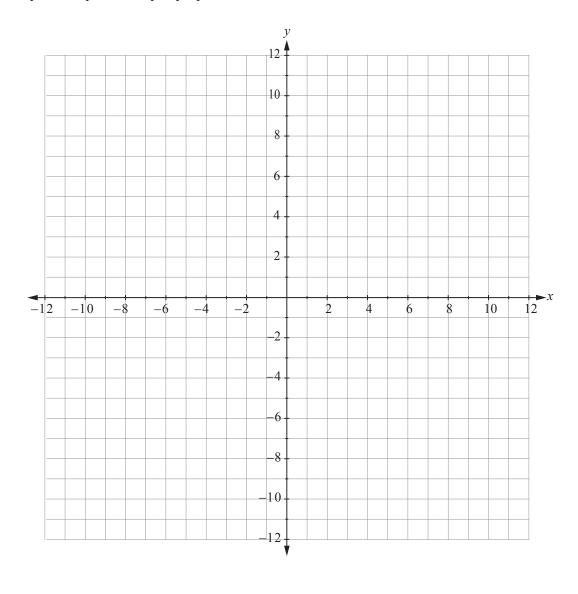
Assessor's use only

QUESTION ONE

Sketch the graph of $\frac{(x-5)^2}{16} + \frac{y^2}{9} = 1.$

Label any intercepts and any asymptotes.

If you need to redraw this graph, use the grid on page 11, 12 or 13.

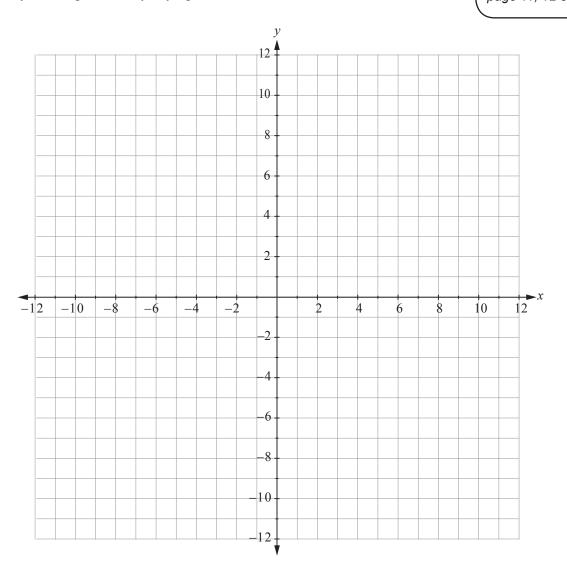


QUESTION TWO

Sketch the graph of $x^2 + y^2 + 10x + 12y + 36 = 0$.

Label any intercepts and any asymptotes.

If you need to redraw this graph, use the grid on page 11, 12 or 13.

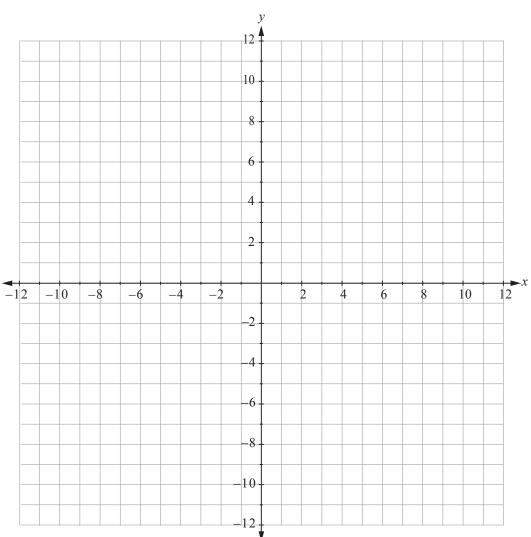


QUESTION THREE

Sketch the graph of the curve defined by $x = 2t^2 + 6$, y = 4t.

Label any intercepts and any asymptotes.

If you need to redraw this graph, use the grid on page 11, 12 or 13.

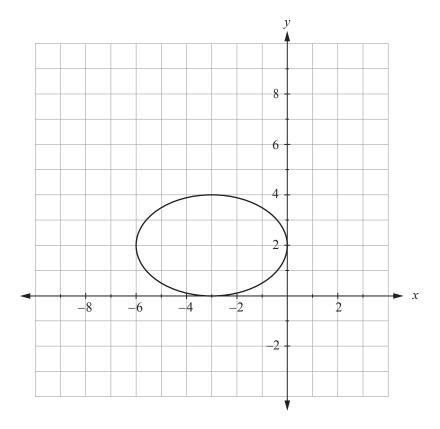


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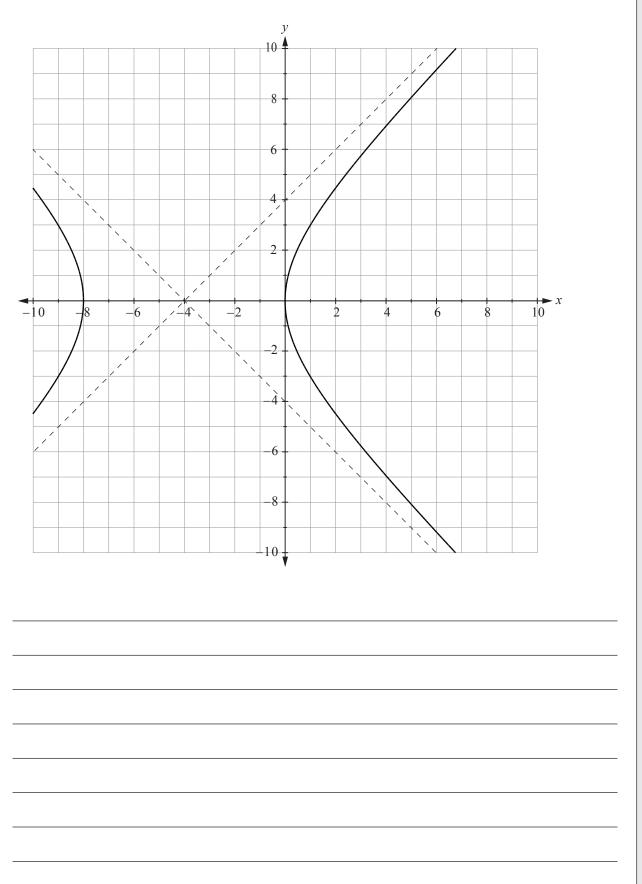
QUESTION FOUR

Assessor's use only

(a) Find the equation of the conic section shown:



(b) Find the equation of the conic section shown:



QUESTION FIVE

Assessor's use only

A curve is given by the equations: $x = 9\cos\theta + 2$ $y = 9\sin\theta$.

Find the equation of the tangent to this curve at the point where $\theta = \frac{\pi}{4}$.				

QUESTION SIX

Assessor's use only

The top shelf of a bookcase has a cross-section as shown in the diagram below.

The inside of the frame is a rectangle measuring 72 cm by 38 cm.

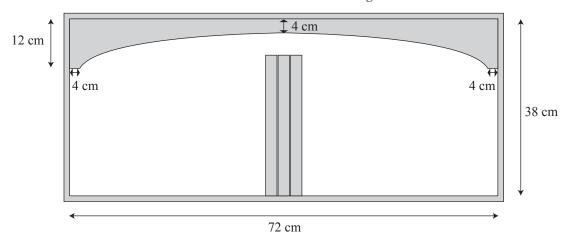
The arch is modelled by a semi-ellipse.

The semi-ellipse is 4 cm down from the top of the frame and 4 cm in from each end.

The major axis of the semi-ellipse is 12 cm down from the top of the frame.

All measurements are taken from the inside edge of the frame.

Diagram not drawn to scale.



Sasha has a set of books. Each book is 30 cm high and 3 cm wide.

How many of these books can stand upright side by side on the shelf?				

QUESTION SEVEN

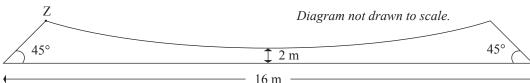
A ramp is being designed for a skateboard park.

At each end of the ramp is a slope, which is at an angle of 45° to the ground.

The cross-section of the ramp is shown in the diagram below.



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The curved part of the cross-section can be modelled by $\frac{y^2}{4} - \frac{x^2}{12} = 1$, y > 0.

where the *x*-axis is at ground level, shown by the base line on the diagram, and the *y*-axis is the axis of symmetry of the curved part.

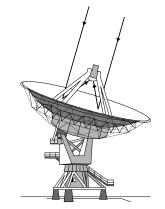
The lowest part of the curve is 2 metres above the base line, which is 16 metres long. Z is the point where the curve and the sloping line meet.

Find the height of the ramp at the point represented by Z on the cross-section.

QUESTION EIGHT

Parabolic antennae are used to collect radio waves.

The diagram below shows a parabola with focus F (0,f) passing through the point P(p,q).



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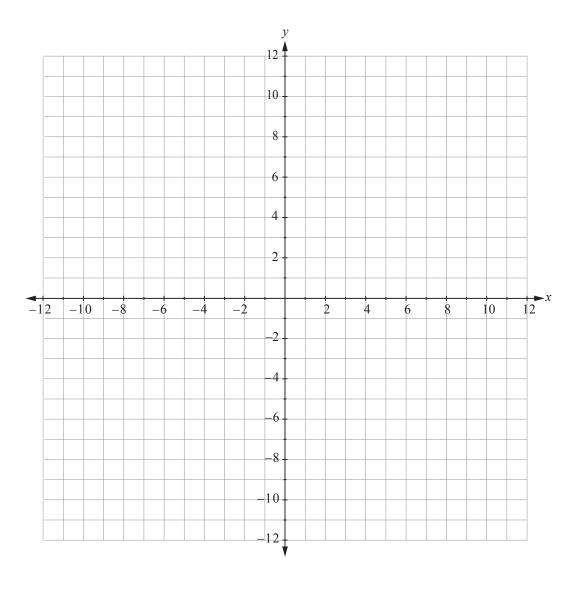
F(0,f) P(p,q)

Write the equation of the tangent at P in the form, ax + by + c = 0, expressing a, b and c in terms of p and q.

If you have made a mistake and need to redraw a graph, use the appropriate copy printed here and clearly number the question.

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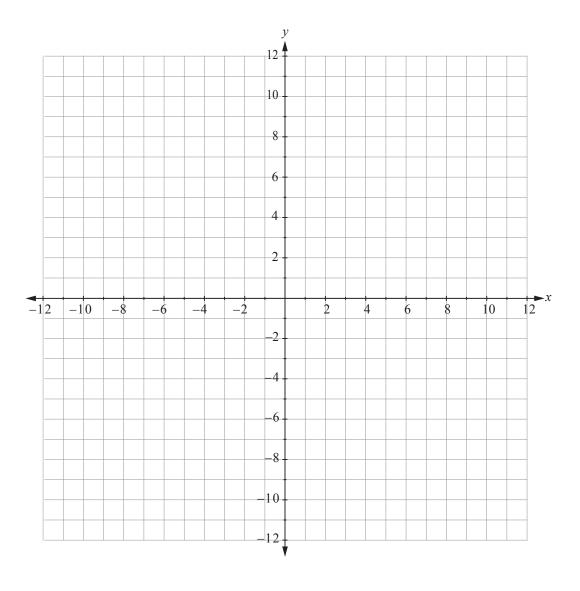
You must cross out the graph that you do not want marked.



If you have made a mistake and need to redraw a graph, use the appropriate copy printed here and clearly number the question.

Assessor's use only

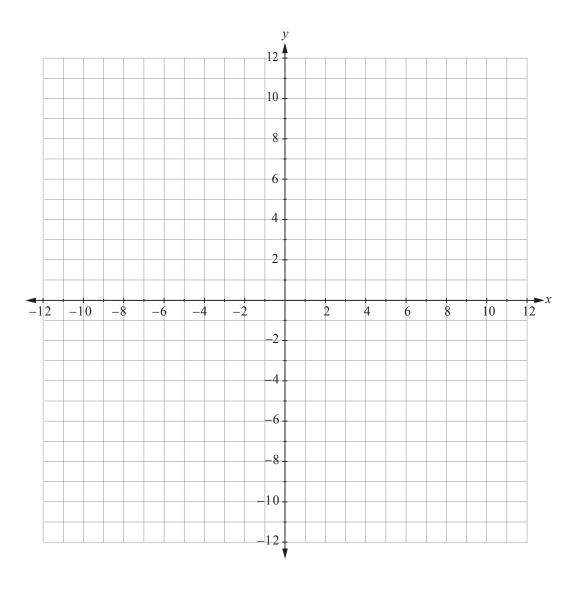
You must cross out the graph that you do not want marked.



If you have made a mistake and need to redraw a graph, use the appropriate copy printed here and clearly number the question.

Assessor's use only

You must cross out the graph that you do not want marked.



Extra paper for continuation of answers if required. Clearly number the question.

Question number	